Outline

- We did an introduction to confidence intervals for the mean:

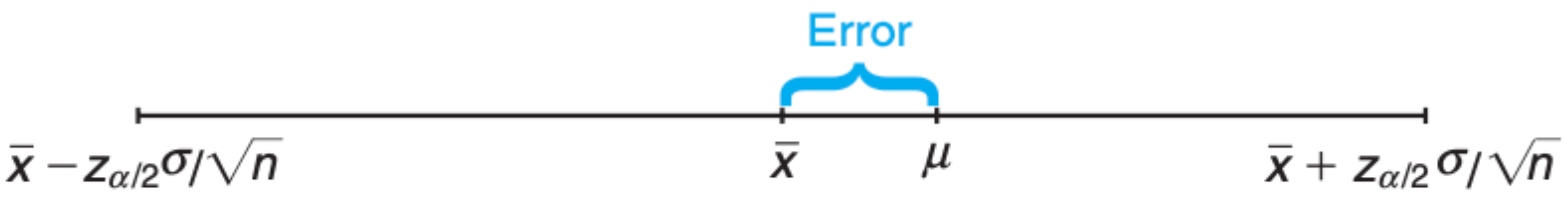
With $100(1-\alpha)$ confidence μ is in $\bar{x} \pm z_{\alpha/2} s.e.(\bar{x})$

Today

- Error in estimating μ
- One-sided confidence intervals for the mean
- Confidence intervals for the mean with σ^2 unknown
  - t-distribution
- Prediction and tolerance intervals (if time permits)
- Confidence intervals for difference of two means (if time permits)

Confidence Interval for the Mean

Error in Estimating μ:



Theorem 9.1: If $\bar{x}$ is used as an estimate of μ, we can be $100(1-\alpha)\%$ confident that the error will not exceed $z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$.

Confidence Interval for the Mean

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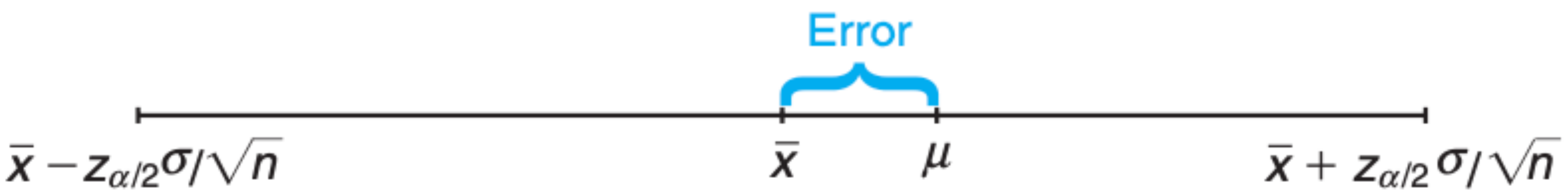

Theorem 9.1: If $\bar{x}$ is used as an estimate of μ, we can be $100(1-\alpha)\%$ confident that the error will not exceed $z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$.

If we need to determine necessary sample size given error e:

Theorem 9.2: If $\bar{x}$ is used as an estimate of μ, we can be $100(1-\alpha)\%$ confident that the error will not exceed a specified amount e when the sample size is

$$n = \left(\frac{z_{\alpha/2}\sigma}{e}\right)^2.$$

Confidence Interval for the Mean

Example 9.3: How large a sample is required if we want to be 95% confident that our estimate of μ in Example 9.2 is off by less than 0.05?

Use $z_{0.025} = 1.96$.

Note $\sigma = 0.3$ in Example 9.2.

Confidence Interval for the Mean

Example 9.3: How large a sample is required if we want to be 95% confident that our estimate of μ in Example 9.2 is off by less than 0.05?

Use $z_{0.025} = 1.96$.

Note $\sigma = 0.3$ in Example 9.2.

Solution:

$$n = \left[\frac{(1.96)(0.3)}{0.05}\right]^2 = 138.3.$$

$\Rightarrow$ a sample size of 139 suffices.

One-Sided Confidence Intervals

One-Sided Confidence Bounds on μ, σ^2 Known	If $\bar{X}$ is the mean of a random sample of size n from a population with variance σ^2, the one-sided $100(1-\alpha)\%$ confidence bounds for μ are given by upper one-sided bound: $\bar{x} + z_\alpha \sigma/\sqrt{n}$; lower one-sided bound: $\bar{x} - z_\alpha \sigma/\sqrt{n}$.

One-Sided Confidence Intervals

Example 9.4: In a psychological testing experiment, 25 subjects are selected randomly and their reaction time, in seconds, to a particular stimulus is measured. Past experience suggests that the variance in reaction times to these types of stimuli is 4 sec^2 and that the distribution of reaction times is approximately normal. The average time for the subjects is 6.2 seconds. Give an upper 95% bound for the mean reaction time.

Use norm.ppf(0.95)=1.645 or norm.ppf(0.975)=1.96

Choose which one to use.

One-Sided Confidence Intervals

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Use norm.ppf(0.95)=1.645 or norm.ppf(0.975)=1.96

Choose which one to use.

Solution:

95 % upper one-sided bound:

$$\bar{x} + z_\alpha \sigma/\sqrt{n} \quad \Rightarrow \quad 6.2 + z_{0.05} 2/\sqrt{25} = 6.2 + 1.645 \times 2/5 = 6.2 + 0.658 = 6.858$$

We are 95 % confident that mean reaction time is less than 6.858.

Confidence Interval for the Mean When σ^2 Unknown

Corollary 8.1: Let $X_1, X_2, \ldots, X_n$ be independent random variables that are all normal with mean μ and standard deviation σ. Let

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \qquad \text{and} \qquad S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2.$$

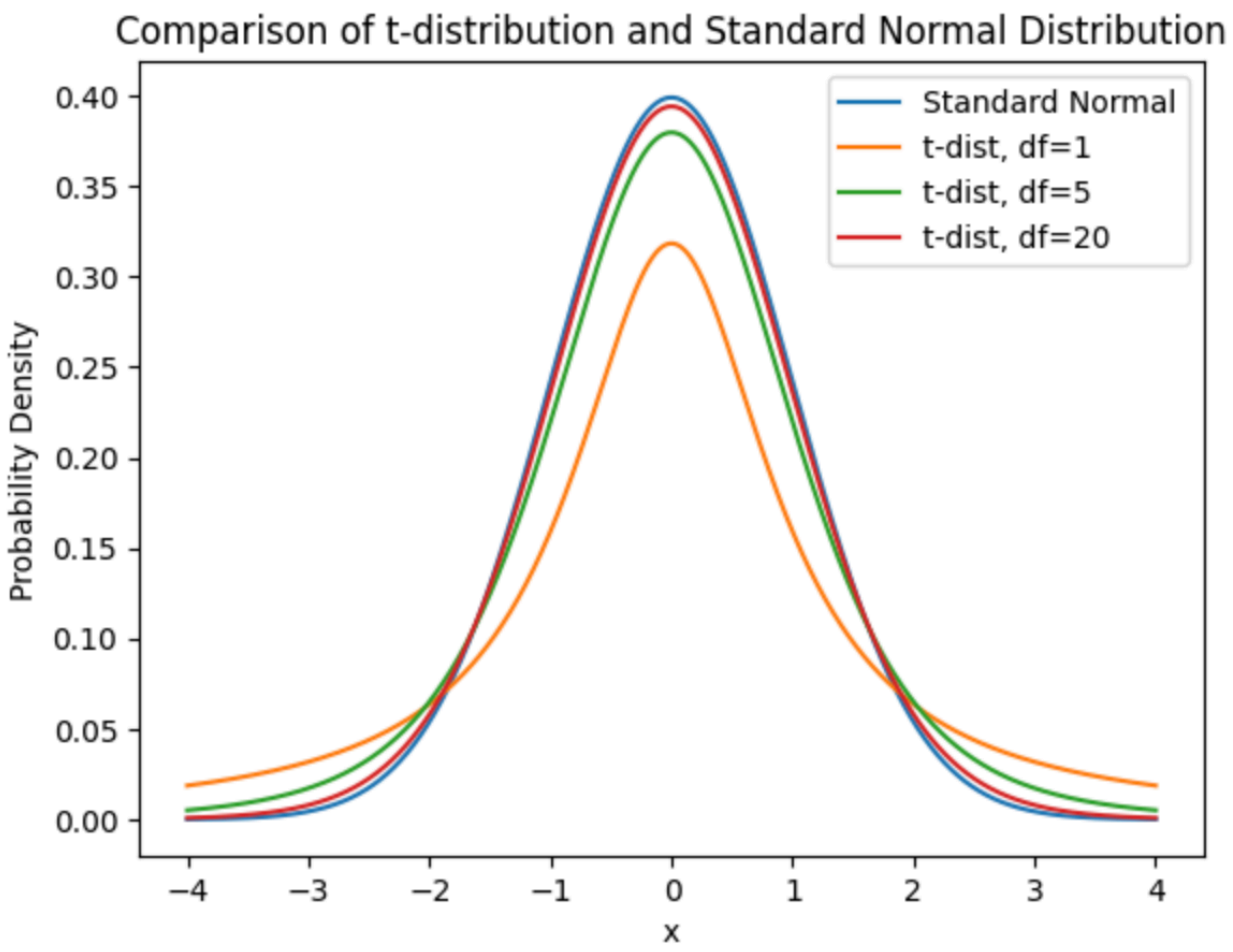
Then the random variable $T = \frac{\bar{X}-\mu}{S/\sqrt{n}}$ has a t-distribution with $v = n - 1$ degrees of freedom.

From Section 8.6 of the textbook.

What is the t-distribution?

Confidence Interval for the Mean When σ^2 Unknown

(Student's)
t distribution




```
from scipy.stats import t
t.cdf(1,20)
```

0.8353717114141455

William S Gosset
(aka *Student*)

While working at
Guinness

Confidence Interval for the Mean When σ^2 Unknown

How to compute the confidence interval?

Almost same as σ known case, except replace z with t and σ with s.

Confidence Interval on μ, σ^2 Unknown

If $\bar{x}$ and s are the mean and standard deviation of a random sample from a normal population with unknown variance σ^2, a $100(1-\alpha)\%$ confidence interval for μ is

$$\bar{x} - t_{\alpha/2}\frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2}\frac{s}{\sqrt{n}},$$

where $t_{\alpha/2}$ is the t-value with $v = n - 1$ degrees of freedom, leaving an area of $\alpha/2$ to the right.

Confidence Interval for the Mean When σ^2 Unknown

Example: Heights of men approximately normally distributed. Sample of size 9: 168, 176, 195, 182, 188, 150, 165, 170, 158. Find $95\,\%$ confidence interval for distribution mean.

Use t.ppf(0.975,8)=2.306

Solution:

Confidence Interval for the Mean When σ^2 Unknown

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Solution:

$95\ \%$ confidence interval:

$$\bar{x} = \frac{1}{9}\sum_{i=1}^{9} X_i = 172.4,\ S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2 = \frac{1}{8}\sum_{i=1}^{9}(X_i - 172.4)^2 = 206.03 \Rightarrow S \approx 14.35$$

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$$\bar{x} \pm t_{\alpha/2} S/\sqrt{n} = 172.4 \pm t_{0.025} \times 14.35/\sqrt{9} = 172.4 \pm 2.306 \times 14.35/3 = 172.4 \pm 11.03$$

which implies $161.37 < \mu < 183.43$.