Outline

Review

- Confidence intervals for the difference of two means: Unpaired/Paired
- Confidence intervals for the variance:

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$$

Today

- Examples on confidence interval for the variance
- Introduction to Hypothesis testing (Chapter 10)

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- ➢ Confidence intervals for the difference of two means: Unpaired/Paired
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$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}$$

this ratio gets close to 1 as degrees of freedom (thus sample size) increases.

Today

- ➢ Examples on confidence interval for the variance
- ➢ Introduction to Hypothesis testing (Chapter 10)

Confidence Interval for the Variance

Example 9.18: The following are the weights, in decagrams, of 10 packages of grass seed distributed by a certain company: 46.4, 46.1, 45.8, 47.0, 46.1, 45.9, 45.8, 46.9, 45.2, and 46.0. Find a 95% confidence interval for the variance of the weights of all such packages of grass seed distributed by this company, assuming a normal population.

Assume with the given values $\frac{1}{n-1}\sum_{i=1}^{10}(X_i - \overline{X})^2 = 0.286$

Use $\chi^2_{0.025}$ with 9 dof as 19.023 and $\chi^2_{0.975}$ with 9 dof as 2.7.

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Use $\chi^2_{0.025}$ with 9 *dof* as 19.023 and $\chi^2_{0.975}$ with 9 *dof* as 2.7.

Solution:

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}} \Rightarrow \frac{9 \times 0.286}{19.023} < \sigma^2 < \frac{9 \times 0.286}{2.7}$$

$$\Rightarrow 0.135 < \sigma^2 < 0.953$$

Confidence Interval for the Variance

9.71 A manufacturer of car batteries claims that the batteries will last, on average, 3 years with a variance of 1 year. If 5 of these batteries have lifetimes of 1.9, 2.4, 3.0, 3.5, and 4.2 years, construct a 95% confidence interval for σ^2 and decide if the manufacturer's claim that $\sigma^2 = 1$ is valid. Assume the population of battery lives to be approximately normally distributed.

Assume with the given values

$$\frac{1}{n-1}\sum_{i=1}^{5}(X_i - \bar{X})^2 = 0.815$$

Use $\chi^2_{0.025}$ with 4 *dof* as 11.143 and $\chi^2_{0.975}$ with 4 *dof* as 0.484.

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$$\Rightarrow 0.29 < \sigma^2 < 6.74$$

$\Rightarrow \sigma^2 = 1$ is valid (consistent with the data)

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$$\Rightarrow 0.29 < \sigma^2 < 6.74$$

a wide interval due to small n.

$\Rightarrow \sigma^2 = 1$ is valid (consistent with the data)

Hypothesis Testing

A **statistical hypothesis** is an assertion or conjecture concerning one or more populations.

Statements about parameter/property θ of a distribution/population:

Average GPA is < 2.8

Probability of heads of a coin is > 0.6

People eat more on weekends than in weekdays.

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One-sided vs Two-sided:

one-sided: $\theta > 3.2$

two-sided: $\theta < -1.5$ or $\theta > 2$

one-sided: $\theta \leq -2$

two-sided: $\theta \neq 2$

$\theta < 2$ or $\theta > 2$

Hypothesis Testing

In hypothesis testing the focus is on two hypotheses.

The Null Hypothesis: H_0

Status quo, assumption believed to be true

Coin in my pocket, probability of heads $p = 0.5$

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The Alternative Hypothesis: H_1

Complement of H_0

Novel finding after research

Coin has probability of heads $p \neq 0.5$

Hypothesis Testing

Analogy with proof by contradiction.

To reject claim H_0 :

- Assume H_0 is True.
- Gather axioms, facts.
- Arrive at an impossible event.

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Arrive at an impossible event.

Prove H_1

One with probability 0.

Hypothesis Testing

Example Proof by Contradiction:

H_0 : There is a highest number. H_1 : There is no highest number.

Hypothesis Testing

Example Proof by Contradiction:

H_0 : There is a highest number. H_1 : There is no highest number.

Assume H_0 is True. Let x be the highest number.

Obtain new number $y = x + 1$.

Impossible event: x is the largest number, but y is bigger than it!

H_0 must be false.

Hypothesis Testing

Analogy with proof by contradiction.

To reject claim H_0 :

Assume H_0 is True.

Gather axioms, facts.

Arrive at an impossible event.

Note:

If we can't arrive at an impossible event, this doesn't imply H_0 is true.

Hypothesis Testing

Analogy with proof by contradiction.

To reject claim H_0 :

Assume H_0 is True.

Gather axioms, facts.

Arrive at an impossible event.

Similarly in hypothesis testing:

To reject H_0 :

Assume H_0 is True.

Gather data.

Show near impossibility of data.

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Analogy with proof by contradiction.

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Similarly in hypothesis testing:

To reject H_0 :

Assume H_0 is True.

Gather data.

Show near impossibility of data.

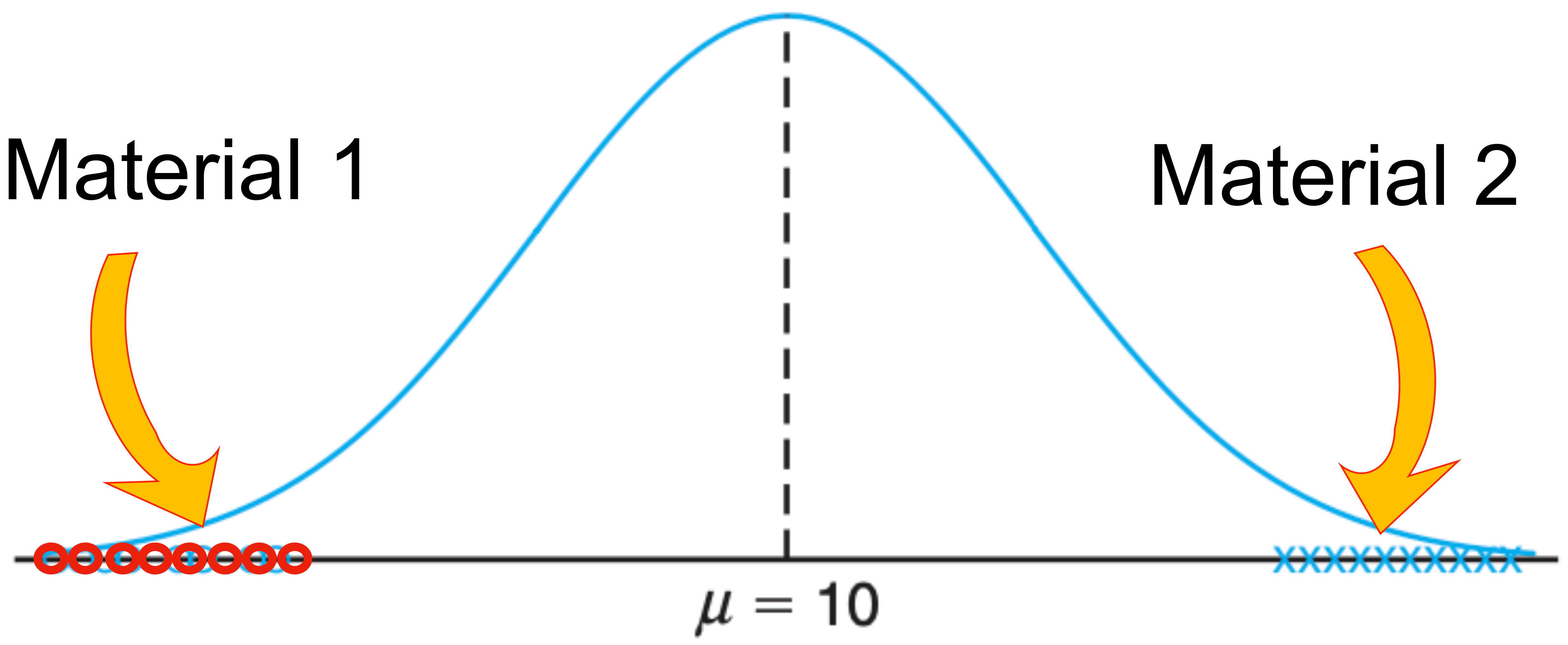
Note:

If the procedure doesn't work we don't reject H_0 (doesn't mean we accept it!).

Hypothesis Testing

Example: Amounts of corrosion using 2 different materials.

$$H_0: \mu_1 = \mu_2 \qquad H_1: \mu_1 \neq \mu_2$$



Probability of obtaining these data is near 0, if H_0 is True.

We can reject H_0 (and accept the alternative hypothesis that the two means are different).

Hypothesis Testing

Two ways of presenting conclusions in hypothesis testing:

Fixed significance level vs P-value

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Fixed significance level vs P-value

<u>Relevant concepts</u>

Test statistic

Critical region/value

Significance level (Size of the test)

Type I error, Type II error

Power of the test

Hypothesis Testing

Two ways of presenting conclusions in hypothesis testing:

Fixed significance level vs P-value

<u>Relevant concepts</u>

Test statistic

Critical region/value

Significance level (Size of the test)

Type I error, Type II error

Power of the test

→ let's first go over these.

Hypothesis Testing

Test statistic:

The result from actual data that we base our decision on. Denoted with X.

Critical Region:

Set of outcomes that are extreme (near impossible) when H_0 is true.

Leads to rejection of H_0.

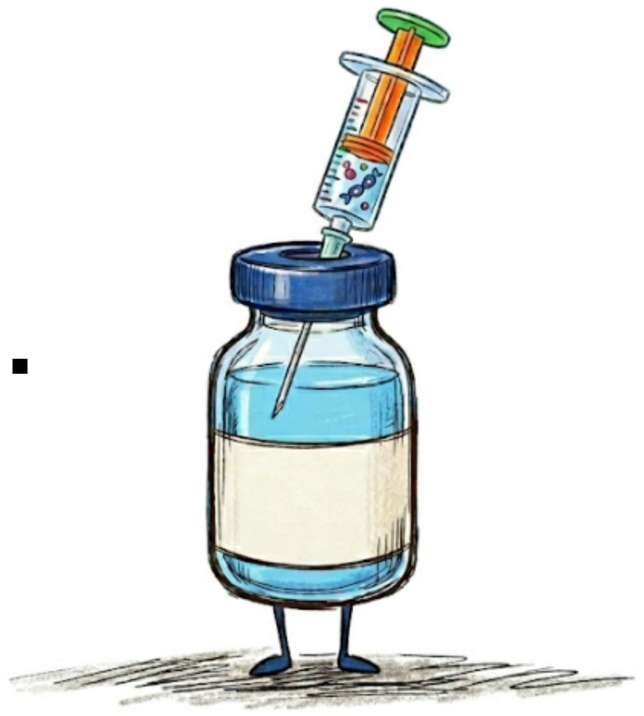
Critical Value:

Boundary line of the critical region.

Hypothesis Testing

Example: One-sided test with a discrete random variable

Testing whether a new vaccine has effectiveness more than $25\ \%$.



$$H_0 : \ p \leq 0.25 \qquad H_1 : \ p > 0.25$$

Critical Region

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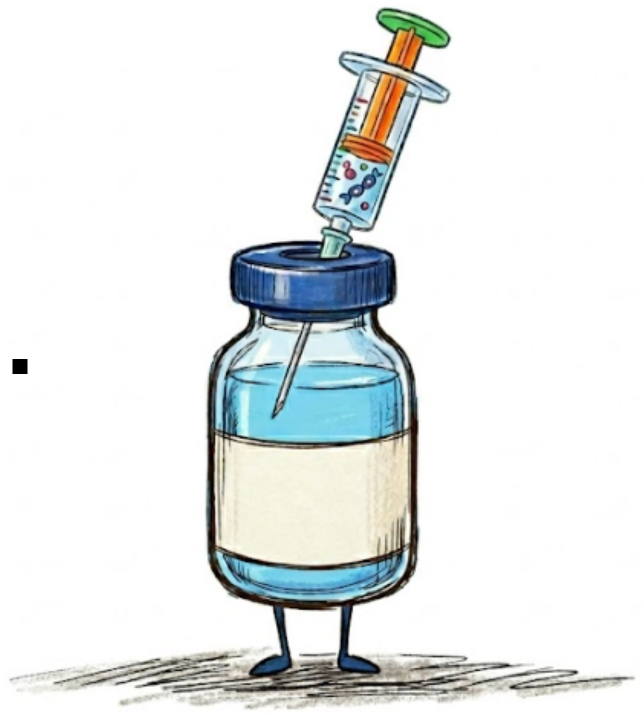


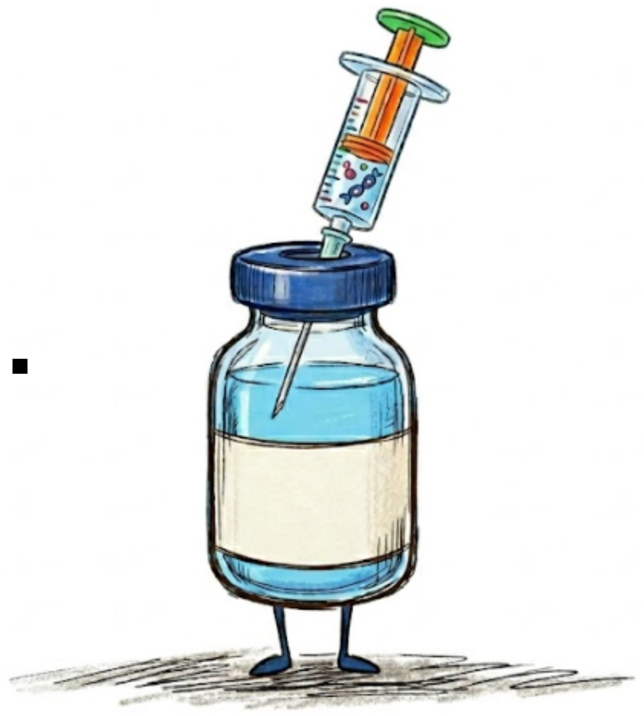
$$H_0 : \ p \leq 0.25 \qquad\qquad H_1 : \ p > 0.25$$

Say we randomly choose a sample of size 20.

x values indicate possible numbers of vaccine successes in any sample data.

Critical Region

Example: One-sided test with a discrete random variable



Testing whether a new vaccine has effectiveness more than $25\ \%$.

$$H_0 :\ p \leq 0.25 \qquad H_1 :\ p > 0.25$$

Say we randomly choose a sample of size 20.

If H_0 were true we'd expect around at most 5 or so many successes.

If we observe a number considerably larger, say > 8 we can reject H_0.

Critical Region

Example: One-sided test with a discrete random variable

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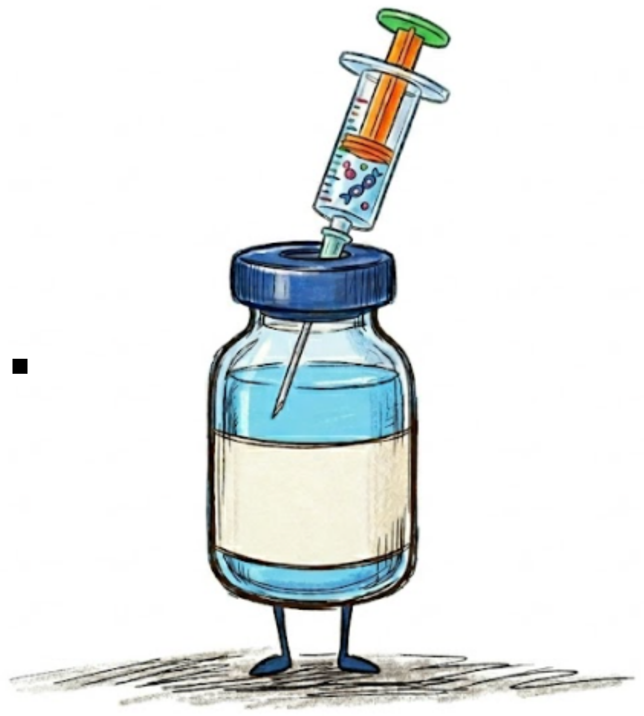

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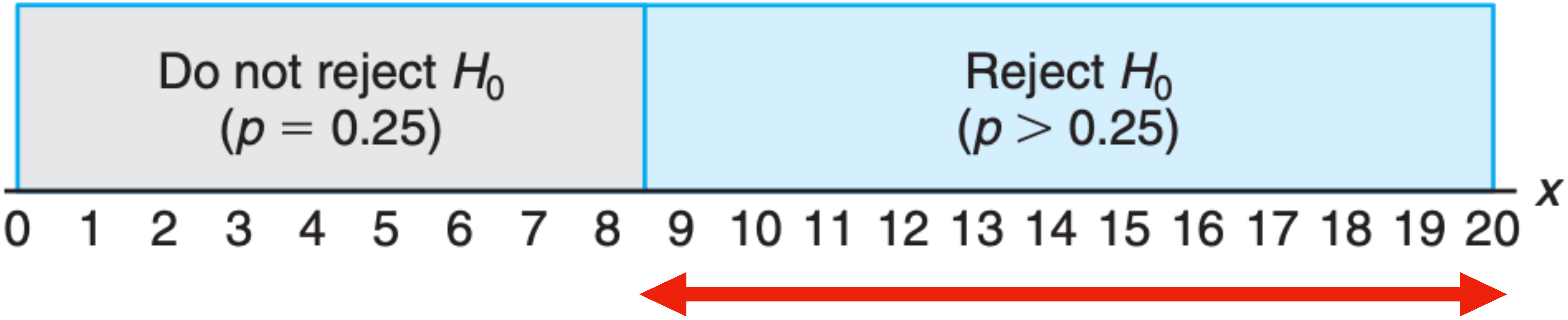

Critical region:

A sample with $x > 8$ leads to rejection of H_0 in favor of H_1

Critical Region

Example: One-sided test with a discrete random variable

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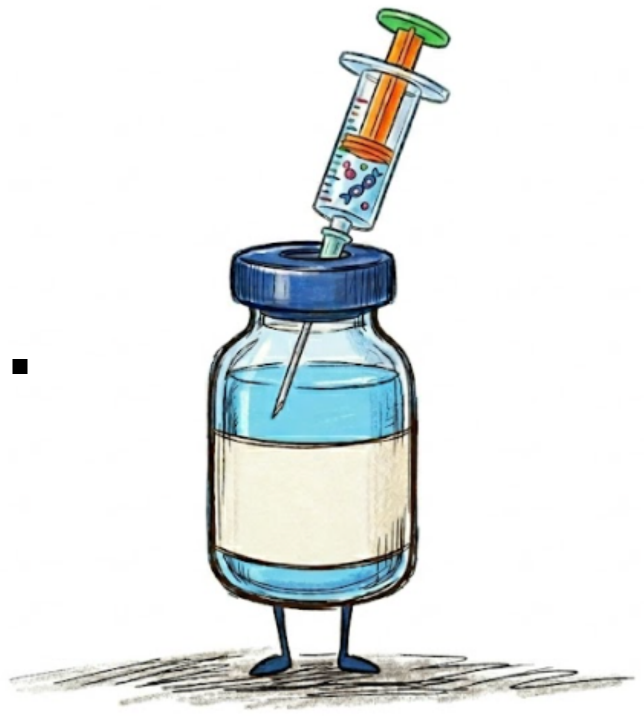

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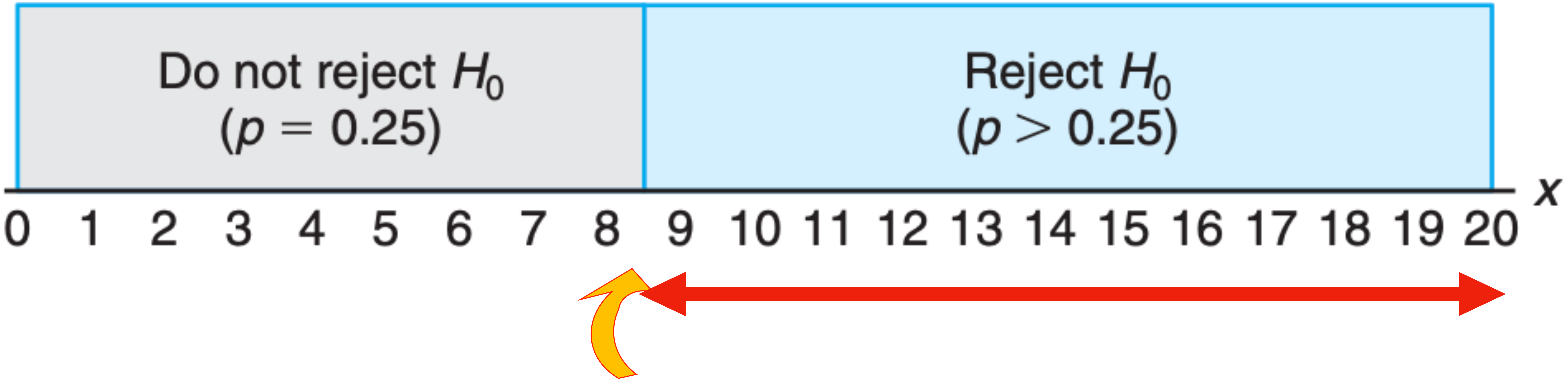

Critical value is 8.

Critical Region

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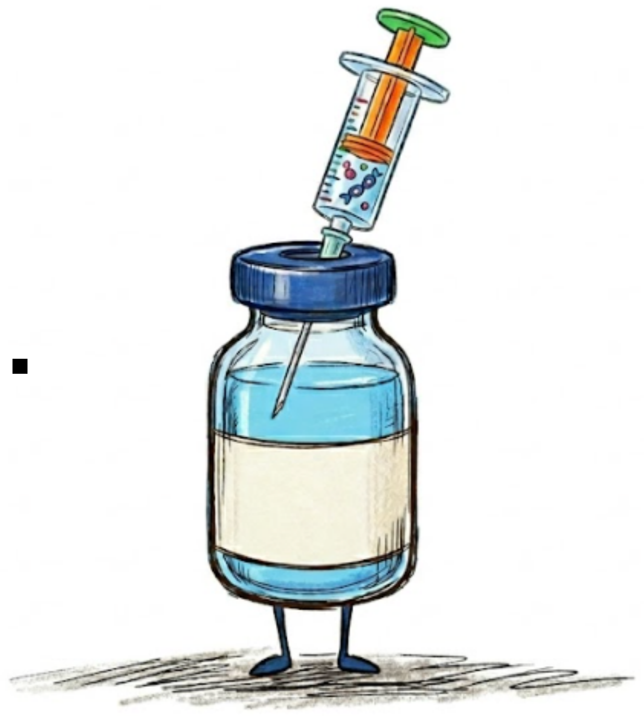

$$H_0: \; p \leq 0.25 \qquad H_1: \; p > 0.25$$

Note: Textbook writes this as $H_0: \; p = 0.25$ (but they still mean $p \leq 0.25$)

Critical Region

Example: Two-sided test with a continuous random variable

Testing whether average weight of male students is 68kg.

$$H_0 : \ \mu = 68 \qquad H_1 : \ \mu \neq 68$$

Critical Region

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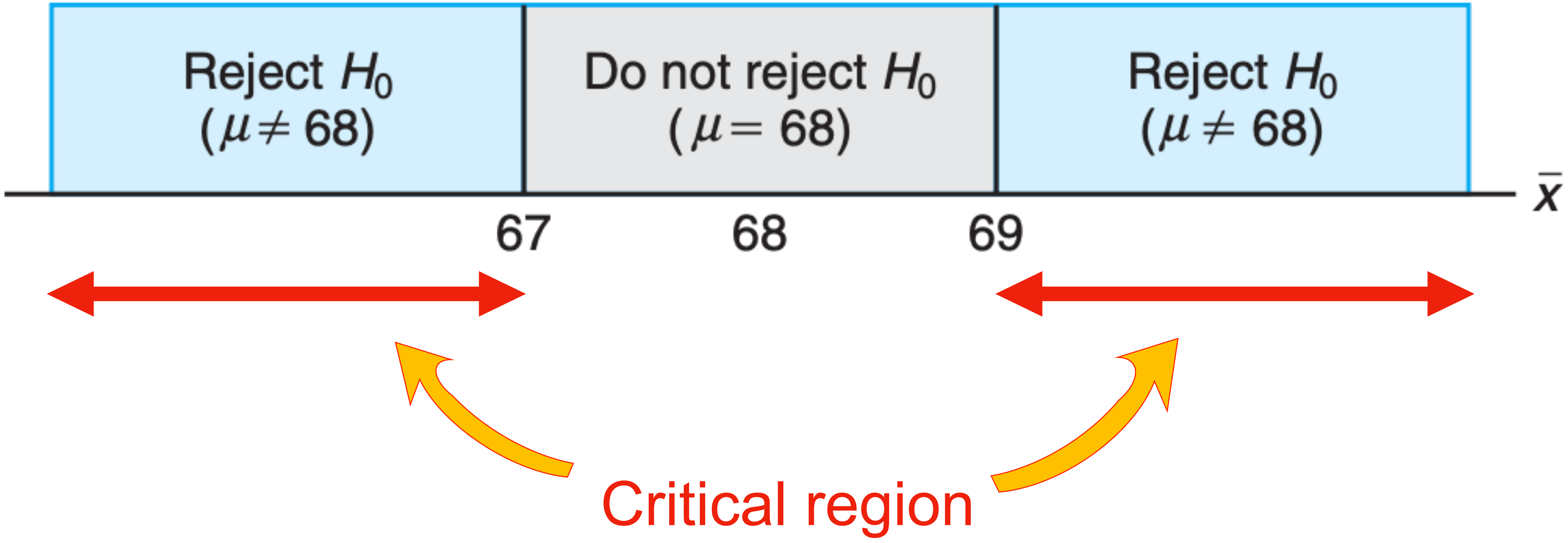

Significance Level and Type I Error

How to decide critical region/value?

We fix a significance level α: Probability of making Type I error in the test.

Rejection of the null hypothesis when it is true is called a **type I error**.

Significance Level and Type I Error

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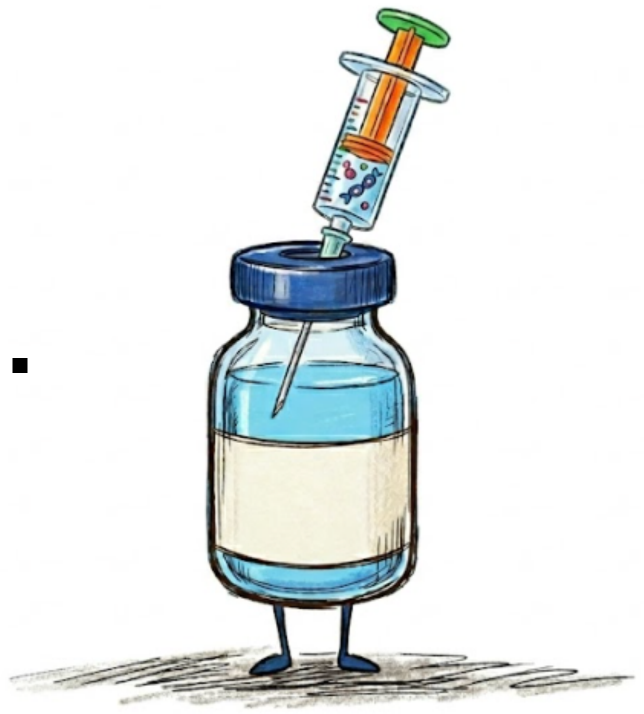

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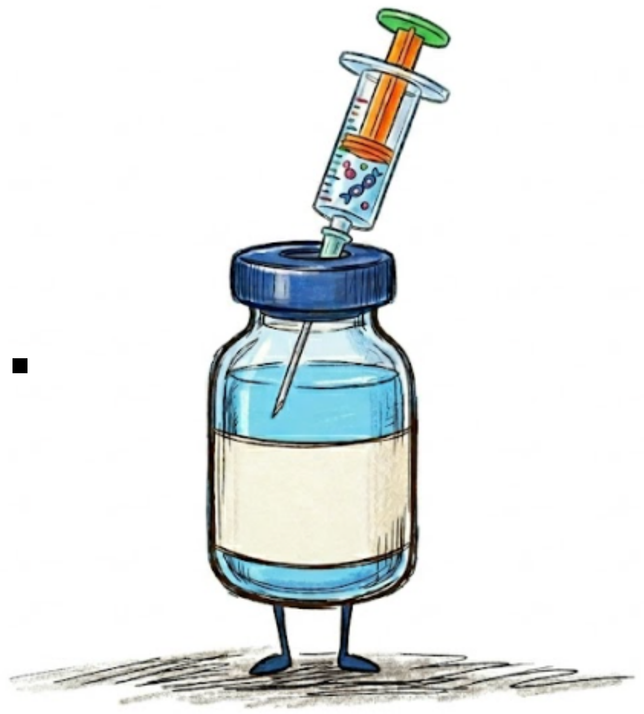

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Earlier we chose critical region $X > 8$ for rejection of H_0 (sample size 20).

$$\alpha = P(\text{type I error}) = P\left(X > 8 \text{ when } p = \frac{1}{4}\right) = \sum_{x=9}^{20} b\left(x; 20, \frac{1}{4}\right)$$

$$= 1 - \sum_{x=0}^{8} b\left(x; 20, \frac{1}{4}\right) = 1 - 0.9591 = 0.0409.$$

Significance Level and Type I Error

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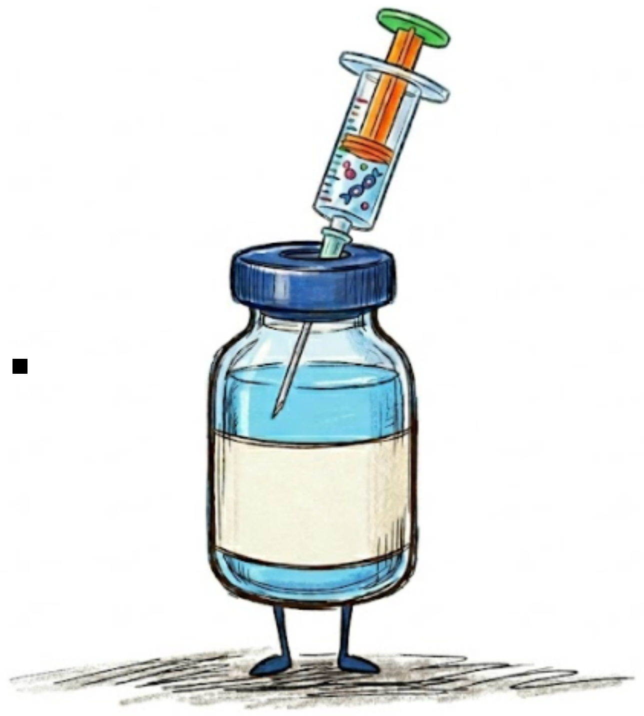

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rejection of H_0

with probability of success $\frac{1}{4}$ (H_0 is True)

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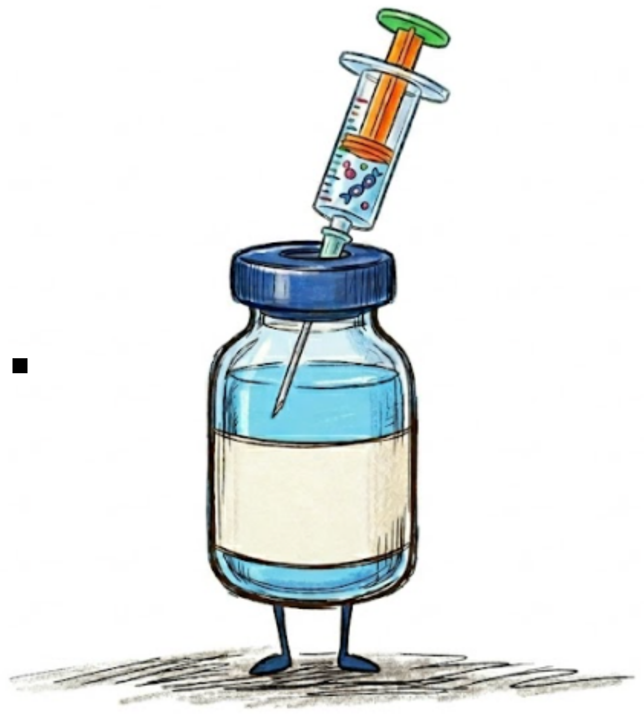

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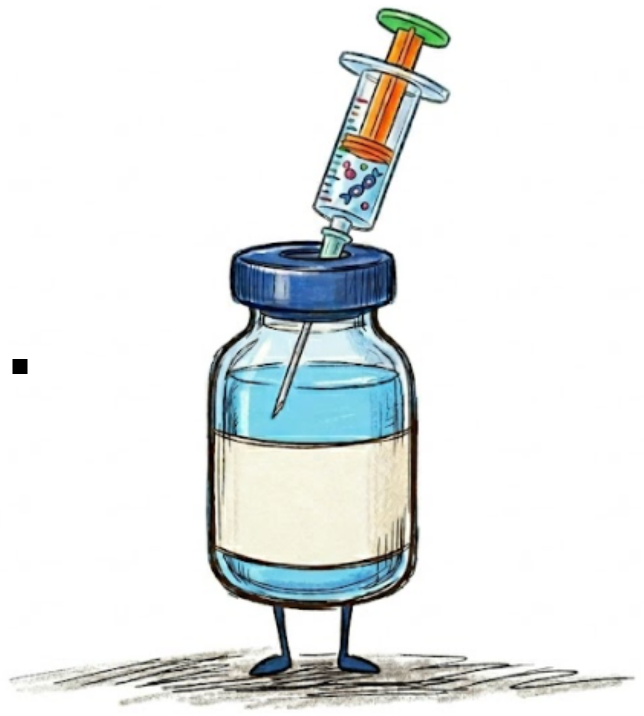

$$H_0 : \ p \leq 0.25 \qquad H_1 : \ p > 0.25$$

Earlier we chose critical region $X > 8$ for rejection of H_0 (sample size 20).

Is this not a problem?

No! If p were smaller we would get even smaller

$$\alpha = P(\text{type I error}) = P\left(X > 8 \text{ when } p = \frac{1}{4}\right) = \sum_{x=9}^{20} b\left(x; 20, \frac{1}{4}\right)$$

say $p = \frac{1}{5}$

$$= 1 - \sum_{x=0}^{8} b\left(x; 20, \frac{1}{4}\right) = 1 - 0.9591 = 0.0409.$$

→ probability of Type I error.

Significance Level and Type I Error

This is just to illustrate the concepts of significance level and Type I error. Normally, we are not given the critical region.

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Significance Level and Type I Error

This is just to illustrate the concepts of significance level and Type I error. Normally, we are not given the critical region.

Instead, we will have fixed α and we will figure out the critical region from it.

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